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It will be found that for this series the following relations are true:

$$u_{i+2}u_{i-1} - u_iu_{i+1} = \pm 1, \qquad u_i^4 - u_{i-2}u_{i-1}u_{i+1}u_{i+2} = 1, \qquad \text{and} \qquad S_i = u_{i+2} - u_2,$$

where S_i is the sum of the first i terms of the series. This latter series has received the names Fibonacci's and Lamé's, and is the subject of several interesting studies. In the *Journal de Mathématiques Elémentaires*(Vuibert), 44e année, p. 11, it is shown that the sum of the squares of two consecutive terms of Fibonacci's series is a term of the series. In the *Intermédiaire des Mathématiciens*, 1916, pages 64 and 599, it is proved that if p is a prime of the form $10n \pm 3$ it is a divisor of u_{p+1}; if of the form $10n \pm 1$ it is a divisor of u_{p-1}. In *Nouvelles Annales de Mathématiques*, 1884, p. 533, is a proof of the theorem that $(2u + 1)^i - u^{3i} = 1$ provided that the exponents in the expansion are replaced by indices and the resulting terms u_n are terms of Fibonacci's series.[1]

Also solved by NORMAN ANNING, J. P. BALLANTINE, E. B. ESCOTT, H. L. OLSON, E. J. OGLESBY, and LOUIS WEISNER.

2811 [1920, 81]. Proposed by J. L. RILEY, Stephenville, Texas.

Given cube roots of 60, 61, 63 and 64, to find the cube root of 62 by the method of differences.

SOLUTION AND REMARKS BY E. B. ESCOTT, Oak Park, Illinois.

Let u_1, u_2, u_3, u_4, u_5 be a series of functions with equidistant arguments. By the method of differences, the first term of the fourth order of difference of u_1 is

$$u_5 - 4u_4 + 6u_3 - 4u_2 + u_1.$$

Let this equal zero and solve for u_3, whence

$$u_3 = \tfrac{1}{6}[4(u_2 + u_4) - (u_1 + u_5)].$$

If

$$u_1 = \sqrt[3]{60} = 3.91486\ 76412\ -, \qquad u_2 = \sqrt[3]{61} = 3.93649\ 71831, \qquad u_3 = \sqrt[3]{62},$$
$$u_4 = \sqrt[3]{63} = 3.97905\ 72079\ - \qquad \text{and} \qquad u_5 = \sqrt[3]{64} = 4,$$

we have by above formula, $\sqrt[3]{62} = 3.95789\ 16538$. This is true to 7 decimals, the value to 10 decimals being $3.95789\ 16097\ -$.

A more general method is to use Newton's Interpolation Formula, where the intervals are not necessarily equal, *i.e.*,

$$u_x = u_a + (x - a)\cdot\delta'(a, b) + (x - a)(x - b)\cdot\delta''(a, b, c) + \cdots, \tag{1}$$

where

$$\delta'(a, b) = \frac{u_a - u_b}{a - b}, \qquad \delta''(a, b, c) = \frac{\delta'(a, b) - \delta'(b, c)}{a - c}, \qquad \cdots.$$

See Thiele, *Interpolationsrechnung*, page 5, or Boole, *Finite Differences*, Chapter 3, Exercise 19. In this problem, $u_{62} = \sqrt[3]{62}$, $u_a = u_{60} = \sqrt[3]{60}$, $u_b = u_{61} = \sqrt[3]{61}$, $u_c = u_{63} = \sqrt[3]{63}$, $u_d = u_{64} = \sqrt[3]{64}$.

Then

$$\delta'(61, 60) = \frac{\sqrt[3]{61} - \sqrt[3]{60}}{61 - 60} = .02162\ 95419,$$

$$\delta'(63, 61) = \frac{\sqrt[3]{63} - \sqrt[3]{61}}{63 - 61} = .02128\ 00124,$$

$$\delta'(64, 63) = \frac{\sqrt[3]{64} - \sqrt[3]{63}}{64 - 63} = .02094\ 27921,$$

$$\delta''(63, 61, 60) = \frac{\delta'(63, 61) - \delta'(61, 60)}{63 - 60} = -\ .00011\ 65098,$$

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[1] The literature of this series has been discussed already in this MONTHLY, *1918*, 234–238, 462, by R. C. Archibald; reference to a recent extended elaboration of these notes was made *1920*, 314.—EDITORS.

$$\delta''(64, 63, 61) = \frac{\delta'(64, 63) - \delta'(63, 61)}{64 - 61} = -\ .00011\ 24068,$$

and

$$\delta'''(64, 63, 61, 60) = \frac{\delta''(64, 63, 61) - \delta''(63, 61, 60)}{64 - 60} = .00000\ 10258.$$

Substituting in Newton's Formula, we have

$$\sqrt[3]{62} = \sqrt[3]{60} + 2 \times .02162\ 95419 + 2 \times 1 \times (-\ .00011\ 65098)$$
$$+ 2 \times 1 \times (-1) \times .00000\ 10258 \quad (2)$$

$$= 3.95789\ 16538 \text{ true to 7 decimals, as in the other solution.}$$

In the above formula, the arguments may be any number, and, in particular, we may have any number of them equal. In this case the "divided differences" become

$$\delta'(a, a) = \lim_{b \doteq a} \frac{u_a - u_b}{a - b} = \frac{du_b}{db}, \qquad \delta''(a, a, a) = \lim_{b \doteq a} \frac{\delta''(a, a) - \delta''(b, b)}{a - b} = \frac{1}{2}\frac{d^2u_b}{db^2},$$

$$\delta'''(a, a, a, a) = \frac{1}{2\cdot 3}\frac{d^3u_b}{db^3} \cdots .$$

Newton's Interpolation formula for the given functions $u_a, u_b, u_c, u_d, u_d, u_d, \cdots$, where the last function may be used an indefinite number of times, becomes

$$u_x = u_a + (x - a)\delta'(a, b) + (x - a)(x - b)\delta''(a, b, c) + (x - a)(x - b)(x - c)\cdot\delta'''(a, b, c, d)$$
$$+ (x - a)(x - b)(x - c)(x - d)\cdot\delta^{\text{iv}}(a, b, c, d, d) + \cdots .$$

In this form, it is a combination of Newton's Interpolation Formula and Taylor's Formula. In the foregoing problem, using $u_{64} = \sqrt[3]{64}$ three times, we have

$$\delta'(64, 64) = \left(\frac{d\sqrt[3]{x}}{dx}\right)_{x=64}\left(\frac{1}{3}x^{-2/3}\right)_{x=64} = .02083\ 33333,$$

$$\delta''(64, 64, 63) = \frac{\delta'(64, 64) - \delta'(64, 63)}{64 - 63} = -\ .00010\ 94588,$$

$$\delta'''(64, 64, 63, 61) = \frac{\delta''(64, 64, 63) - \delta''(64, 63, 61)}{64 - 61} = .00000\ 09827,$$

$$\delta^{\text{iv}}(64, 64, 63, 61, 60) = \frac{\delta'''(64, 64, 63, 61) - \delta'''(64, 63, 61, 60)}{64 - 60} = -\ .00000\ 00108,$$

and

$$\delta^{\text{v}}(64, 64, 64, 63, 61, 60) = .00000\ 00001.$$

This gives us two more terms in the series (2), viz.,

$$+ 2 \times 1 \times (-1)(-2)(-\ .00000\ 00108) + 2(1)(-1)(-2)(-2)(.00000\ 00001)$$
$$= -\ .00000\ 00440.$$

Then $\sqrt[3]{62} = 3.95789\ 16538 - .00000\ 00440$
$= 3.95789\ 16098$, true to 9 decimals.[1]

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[1] The same result may be obtained as follows: In the first part of the discussion above,

$$\Delta_4 = u_5 - 4u_4 + 6u_3 - 4u_2 + u_1$$

was assumed zero, but a correction may be obtained by applying to the terms of Δ_4 the development of u_x in powers of $x - 64$, x being 64 for u_5, 63 for u_4, etc. Then

$$\Delta_4 = u_5^{\text{iv}} - 2u_5^{\text{v}} + \frac{13}{6}u_5^{\text{vi}} - \cdots,$$

where

$$u_5^{(n)} = \left.\frac{d^n\sqrt[3]{x}}{dx^n}\right]_{x=64}.$$

Hence

$$\sqrt{62} = u_3 = \frac{1}{6}[4(u_2 + u_4) - (u_1 + u_5)] + \frac{\Delta_4}{6}.$$

Note. Newton's Interpolation Formula is equivalent to the well-known Euler-Lagrange Interpolation Formula, but is frequently more convenient to use. As shown in the above example, if the interpolation is not sufficiently accurate, we need only add more terms to Newton's Formula, while the Euler-Lagrange Formula will need to be entirely recomputed.

Also solved by Norman Anning, E. J. Oglesby, Arthur Pelletier, J. B. Reynolds, C. C. Wylie, and the Proposer.

NOTES AND NEWS.

It is to be hoped that readers of the Monthly will coöperate in contributing to the general interest of this department by sending items to H. P. Manning, Brown University, Providence, R. I.

Associate Professor E. J. Moulton, of Northwestern University, has been promoted to a full professorship of mathematics.

Professor E. G. Bill, of Dartmouth College, has been appointed to the newly created office, dean of freshmen. He will give up all teaching.

At their Alma Mater, Wellesley College, Miss Rachel Blodgett, recently of Radcliffe College, and Miss Ruby Willis, have been appointed instructors of mathematics.

At the University of Pennsylvania, Assistant Professors G. G. Chambers and H. H. Mitchell have been promoted to full professorships.

Mrs. Theron Clark, of Brown University, has been appointed instructor of mathematics at Bucknell University, Lewisburg, Pa.

Dr. F. D. Murnaghan, of The Johns Hopkins University, has been promoted to be an associate professor of applied mathematics.

Associate Professor W. A. Wilson, of Yale University, has been promoted to a full professorship.

Instructors C. S. Doan and F. H. Hodge, of Purdue University, Lafayette, Ind., have been promoted to assistant professorships.

Mr. L. E. Ward, of Harvard University, has been appointed instructor of mathematics at the Rice Institute, Texas.

Miss May B. Carter, of Brown University, has been appointed instructor of mathematics in The Western College for Women, Oxford, O.

Professor E. R. Smith, of Pennsylvania State College, has been appointed head of the department of mathematics at Iowa State College, Ames, Ia.

Dr. J. R. Ritt, of Columbia University, has been promoted to an assistant professorship of mathematics.

At Princeton University, Mr. Philip Franklin, who received his doctorate there in June, has been appointed instructor of mathematics.

Dr. J. E. Rowe, recently of the Aberdeen Proving Ground, has been appointed head of the department of mathematics at College of William and Mary, Williamsburg, Va. Compare *1920*, 280.

In this case the correction $\Delta_4/6 = -.00000\ 00441 +$ and the value of $\sqrt[3]{62}$ is given to 9 places of decimal as above.—Editor.